HILTI METHOD FOR ANCHOR DESIGN IN UNGROUTED STAND-OFF CONNECTIONS

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1. INTRODUCTION

1.1 Construction

Ungrouted stand-off connections are almost exclusively leveled using nuts below the steel plate. Cast-in-place or post-installed anchors are first installed in the concrete. Post-installed mechanical anchors that rely on torquing or displacement to complete anchor installation generally must first be clamped to the concrete surface at the appropriate torque/displacement.

Leveling nuts and their accompanying washers are threaded and placed onto the rod to roughly to the location of where the base plate will need to sit, as shown in Figure 1. The steel member is then seated onto the leveling nuts and washers, most often with assistance from a crane, and the location of the leveling nuts are adjusted to meet the proper member plumbness and other geometric requirements.

After the steel member is appropriately leveled, top nuts and washers are placed onto the plate and the nuts are tightened, such as by using the turn-of-the-nut method. Examples of an ungrouted stand-off connection during seating and in service are shown in Figure 1.

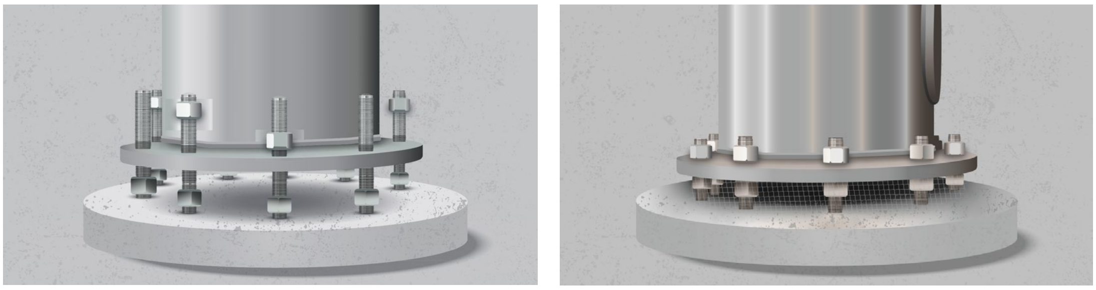

Figure 1. Ungrouted stand-off connection: during seating on leveling nuts (left) and in service (right)

1.2 Structural behavior

1.2.1 Steel resistance of anchors in ungrouted connections

Steel resistance of anchors in ungrouted connections is a function of the combination of shear, bending, and normal force acting on the exposed portion of the anchor. Figure 2 shows the load path of a wind load acting on a sign held by a mast arm through to the concrete in an ungrouted stand-off connection, from the global loads traveling to the connection (left), to the transfer of connection-level forces into individual anchor forces (middle), to the transfer of individual anchor forces through the exposed portion of an anchor through shear, axial, and bending moment forces (right).

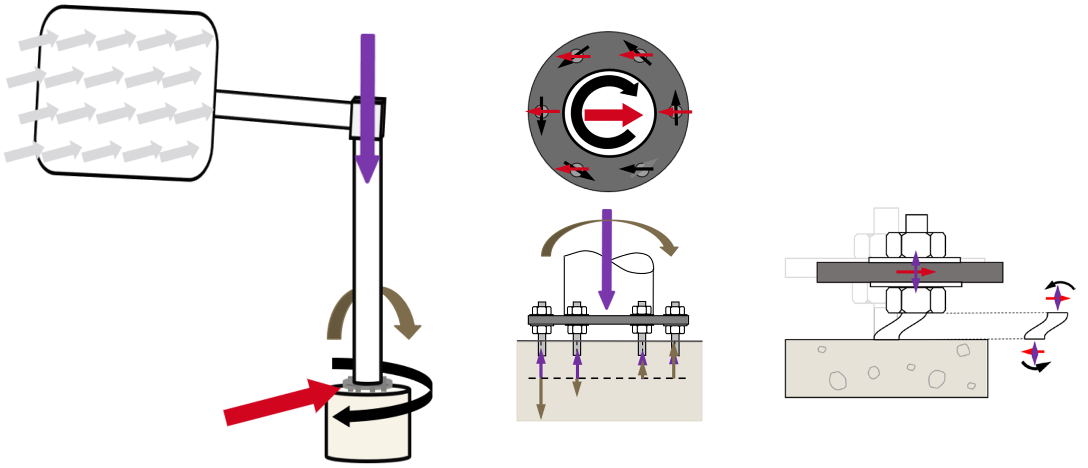

Figure 2. Loads acting on a base connection (left), force transfer to anchors in ungrouted stand-off connections (middle; plan and profile views shown), and forces acting on exposed portion of anchor (right; double curvature shown)

Depending on the boundary conditions, anchors may either be in single or double curvature as illustrated in Figure 3. Double curvature ($\alpha_M = 2.0$) may be assumed when the steel plate is restrained from rotation (generally by other anchors in the connection resisting the rotation), the plate is thick enough to restrain the bending moments at the plate level, and there is a rigid connection between the anchor and the plate (e.g., via clamped leveling and top nuts and/or welds). Where these conditions cannot be confidently met, single curvature ($\alpha_M = 1.0$) should be assumed or further analysis should be performed to determine the correct value of α_M between 1.0 and 2.0.

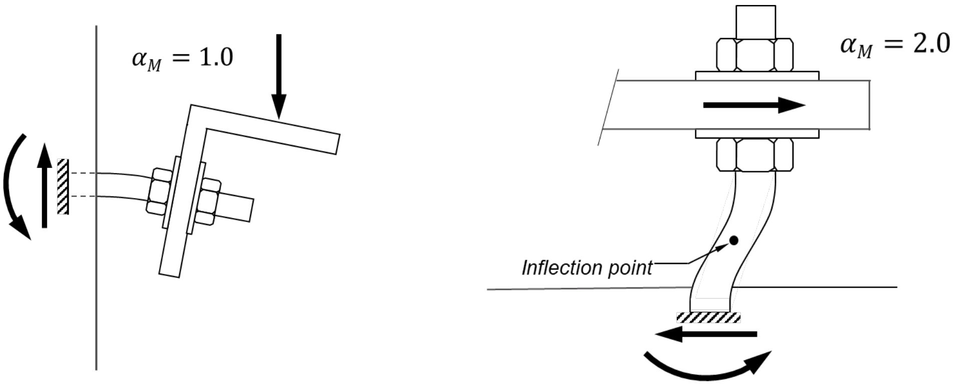

Figure 3. Anchor in single curvature (left) and double curvature (right)

McBride [3] found that the interaction between shear force, normal force, and bending moment in ungrouted stand-off connections can be expressed as described in Sections 2.2.2 and 2.2.4, which ultimately is a reorganization of a three-way interaction equation given in McBride [3]. For anchors in combined tension and shear, the interaction equation in Eq. (1) is conservative, as it ignores beneficial second-order effects where the tensile force relieves the bending moment by acting on the displaced shape in shear. However, because nearly all anchor groups will have combinations of anchors in tension and compression, it is realistic to ignore the beneficial second-order effects on the anchors in tension.

$$\left(\frac{N}{N_o}\right)^2 + \left(\frac{V}{V_o}\right)^2 + \left(\frac{M}{M_o}\right) \leq 1 \qquad (1)$$

where

N = normal force acting on section (tension or compression)

V = shear force acting on section

M = bending moment acting on section $= \frac{V l_a}{2}$

N_o = ultimate uniaxial normal resistance of circular section

V_o = ultimate shear resistance of circular section

M_o = ultimate moment resistance of circular section $= W_{el}\sigma_o = \frac{4r^3 f_{uk}}{3}$

l_a = effective exposed length considered for bolt bending resistance

W_{el} = elastic section modulus of the threaded portion of the anchor; should be taken in relation to the net tensile area of the anchor

r = radius corresponding to net tensile area

In addition, McBride [3] verified that it is appropriate to consider bending between an assumed spall depth (generally taken as 0.5 anchor diameters) below the concrete surface to the bottom of the leveling nut. This is because the predominating curvature in an ungrouted stand-off connection occurs in the exposed portion of the threaded rod due to the high ratios of relative bending stiffness of the nut and the steel plate.

1.2.2 Concrete resistance of anchors in stand-off connections

Concrete breakout resistance in tension is assumed to be unaffected in stand-off conditions.

Concrete edge breakout forces in shear, however, may be amplified by the displacement of the anchor and additional moment traveling through the connection. Figure 4 illustrates the bending moment that acts on the concrete edge breakout body in an ungrouted stand-off anchor. This bending moment adds to the bearing pressure due to shear on the concrete and must be accounted for to properly describe behavior. Appendix A provides the basis for the factor presented in Section 2.3.2.

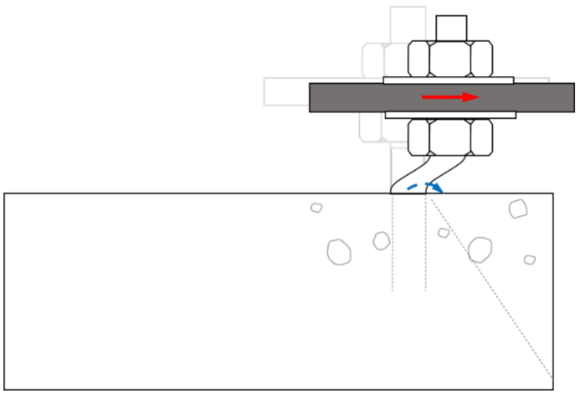

Figure 4. Forces acting on exposed portion of anchor (double curvature shown)

2. DESIGN METHODS

2.1 General

2.1.1 Comparison between EN 1992-4 and Hilti Method

Hilti PROFIS Engineering offers two solutions for the design of anchors in ungrouted stand-off connections: design compliant with Eurocode EN 1992-4 [1] and the Hilti Solutions for Fastenings (SOFA) Method.

EN 1992-4 design of anchors in ungrouted stand-off connections makes conservative assumptions about the lever arm for bending, the interaction between shear and normal forces, and the bending resistance of the anchor section. The Hilti SOFA Method provides a less conservative design approach while also offering a more complete solution than EN 1992-4. The Hilti SOFA Method is recommended in cases where EN 1992-4 does not provide a viable solution.

The primary differences between Eurocode design in accordance with EN 1992-4 [1] design and Hilti SOFA Method are as follows:

	Eurocode design	SOFA design
Load distribution	Forces are distributed to anchors in ungrouted stand-off connections with the assumption that the stiffness in tension and compression is identical.	
Definition of lever arm for bending calculations	Distance from the middle of the steel plate to the reaction point in the concrete. (See Figure 5.)	Distance from the bottom of the leveling nut to the reaction point in the concrete. (See Figure 5.)

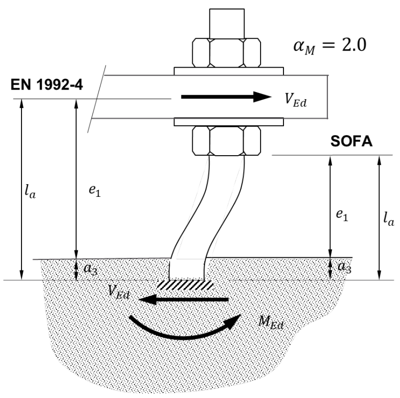

Figure 5. Differences in lever arm definitions between EN 1992-4 and Hilti SOFA Method

	Eurocode design	SOFA design
Shear steel resistance	*See Section 2.2.2.* Bolt bending verification per [1] Eq. (7.37).	*See Section 2.3.2.* Bolt bending and shear verification per McBride [3].
Steel interaction	*See Section 2.2.3.* Steel interaction between axial and shear forces is satisfied directly in the determination of shear steel resistance using [1] Eq. (7.37), which produces a linear relationship between bending and axial capacity.	*See Section 2.3.3.* Interaction between bending, shear, and normal forces per McBride [3].
Concrete failure modes in shear	*See Section 2.2.5.* Concrete edge breakout verification in case of standoff should consider the additional effect of an overturning moment per [1] Section 7.2.2.5 [1].	*See Section 2.3.5.* Concrete edge breakout per [1] Section 7.2.2.5, and the effect of the additional reduction factors is added to the verification, $\psi_{b,u}$, to account for the overturning moment.

Concrete interaction	*See Section 2.2.6.* Interaction between tension and shear concrete failure modes per [1] Table 7.3.	*See Section 2.3.6.* Interaction between tension and shear concrete failure modes per [1] Table 7.3.
Minimum edge distance	Larger of $10h_{ef}$ and $60d$ from the edge in accordance with [1] Section 7.2.2.5.	Minimum edge distance per cover and product requirements.

The Hilti SOFA Method is based on recommendations by McBride [3] that are planned for future incorporation into *fib* Bulletin 58, EN 1992-4, ACI 318, and other relevant anchor design documents.

Section 2.2 provides design procedures for EN 1992-4, and Section 2.3 provides design procedures for the Hilti SOFA Method for steel and concrete failure modes. Provisions applicable to both methods are provided in Sections 2.1.2 through 2.1.4.

2.1.2 Degree of curvature for bending calculations

For both methods, double curvature ($\alpha_M = 2.0$) should only be assumed when the following conditions are met:

1. The steel plate is restrained from rotation (generally from other anchors in the connection aligned in the direction of bending).
2. The steel plate is thick enough to restrain the bending moments at the plate level, $M_{pl,a} = V_{Ed} \cdot l^*$.
3. There is a rigid connection between the anchor and the plate (e.g., via clamped leveling and top nuts and/or welds).

In determining the moment demand on the steel plate in a double-curvature connection, $M_{pl,a}$,

l^*	=	distance from the point of inflection of the anchor in double curvature to the centerline of the steel plate
	=	$l_a/2$ using EN 1992-4 (See Figure 7)
	=	$l_a/2 + a_4$ using the SOFA method (See Figure 9)
a_4	=	distance from the underside of the leveling nut to the centerline of the steel plate (See Figure 9.)

2.1.3 Buckling considerations

Where anchors in compression have length l_a greater than $3d$, it is advised that buckling resistance of the l_a portion of the anchor be verified for both EN 1992-4 and Hilti Method design.

2.1.4 Sectional force distribution

In PROFIS Engineering, forces are distributed to anchors in ungrouted stand-off connections with the assumption that the stiffness in tension and compression is identical.

2.2 EN 1992-4 Design

2.2.1 Axial steel resistance

Axial steel design resistance, $N_{Rk,s}$, is determined in accordance with EN 1992-4 Section 7.2.1.3.

2.2.2 Steel shear with lever arm

In EN 1992-4 provisions, the steel resistance of anchors in grouted stand-off connections is given in 7.2.2.3.2. EN 1992-4 Eq. (7.37) is given as Eq. (2) below. Figures 6 and 7 illustrate the variables that are built into the calculation of Eq. 1 for anchors in single curvature and double curvature, respectively. See also the general provisions of Section 2.1.3 for conditions applicable to double curvature, including the moment demand on the steel plate.

$$V_{Rk,s,M} = \frac{\alpha_M \cdot M_{Rk,s}}{l_a} \qquad (2)$$

where

α_M = 1.0 (single curvature) or 2.0 (double curvature) as determined by the designer

$M_{Rk,s}$ = bending resistance of anchor accounting for the presence of normal force

= $M^0_{Rk,s}(1 - N_{Ed}/N_{Rd,s})$

$M^0_{Rk,s}$ = bending resistance of the anchor taken from the relevant Technical Product Specification and generally taken as $1.5W_{el} \cdot f_{uk}$

W_{el} = elastic section modulus of the threaded portion of the anchor; should be taken in relation to the net tensile area of the anchor

l_a = effective exposed length considered for bolt bending resistance

= $e_1 + a_3$ (see illustration in Figures 6 and 7)

e_1 = distance between the concrete surface and the centerline of the steel plate as pictured in Figures 7 and 8.

a_3 = $0.5d$ where clamping at the concrete surface is not present

= 0 where clamping at the concrete surface is present

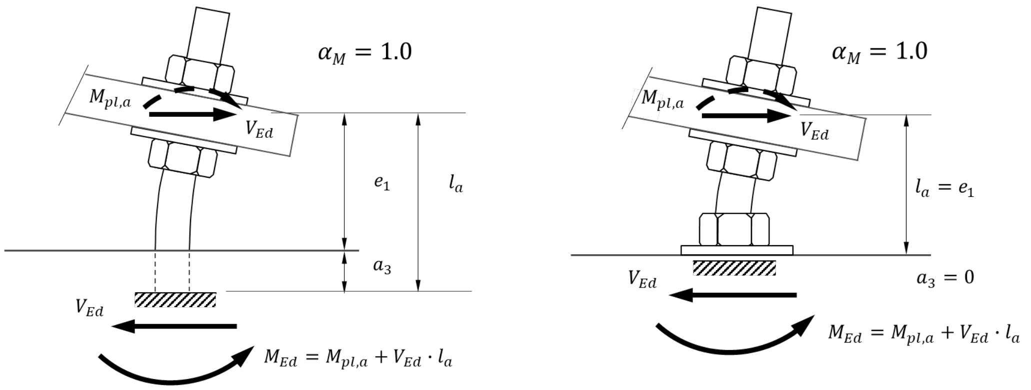

Figure 6. Illustration of dimensions for anchors in single curvature: without clamping nut (left) and with clamping nut (right)

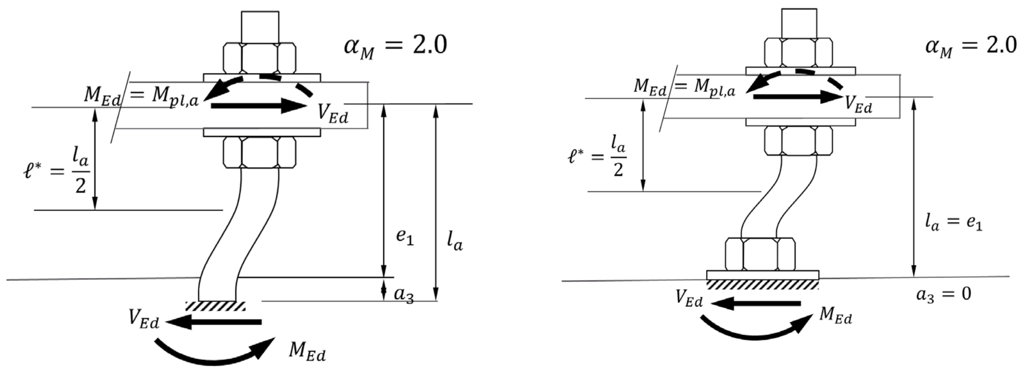

Figure 7. Illustration of dimensions for anchors in double curvature: without clamping nut (left) and with clamping nut (right)

2.2.3 Interaction of shear and axial forces for steel failure

When designing for bending using EN 1992-4 Eq. (7.37), the interaction of shear and axial forces is satisfied directly and is represented as a linear relationship between bending and axial force.

2.2.4 Concrete failure modes in tension

Tensile concrete failure modes described in EN 1992-4, 7.2.1 (cone, pull-out, combined pull-out and concrete, concrete splitting, and concrete blow-out failure) are determined for ungrouted stand-off connections in the same manner as for other connections without modification.

2.2.5 Concrete failure modes in shear

Shear pryout capacity of ungrouted stand-off connections remains identical to that in EN 1992-4 Section 7.2.2.4 whether Eq. (7.36) or Eq. (7.37) are used for anchor steel shear capacity.

However, when designing for bending using EN 1992-4 Eq. (7.37), design is restricted to a minimum edge distance of the larger of $10h_{ef}$ and $60d$ in accordance with EN 1992-4 Section 7.2.2.5. For edge distances larger than this value, shear concrete edge breakout resistance is not required to be calculated. Where closer edge distances are needed, the EN 1992-4 does not offer a solution and it is recommended to use the Hilti SOFA Method.

2.2.6 Interaction of shear and axial forces for concrete failure

Interaction between tension and shear concrete failure modes per EN 1992-4 Table 7.3 and shall satisfy either Eq. (7.55) or Eq. (7.56). Where supplementary reinforcement is present, EN 1992-4 Section 7.2.3.2 applies.

2.3 Hilti SOFA Method Design

2.3.1 Axial steel resistance

Axial steel resistance, $N_{Rk,s}$, is determined in accordance with EN 1992-4, 7.2.1.3.

2.3.2 Steel shear failure of fastener with lever arm

Eq. 3 expresses the shear resistance of an ungrouted stand-off anchor when incorporating the bending moments acting on the exposed portion of the anchor. Figures 8 and 9 show the variables that are built into the calculation of Eq. (3) for anchors in single curvature and double curvature, respectively.

$$V_{Rk,s,M} = \left(\sqrt{{\alpha_{s,M}}^2 + 1} - \alpha_{s,M}\right) \cdot V_{Rk,s} \leq V_{Rk,s} \quad (3)$$

where

$V_{Rk,s}$	=	characteristic shear resistance
$\alpha_{s,M}$	=	$\frac{1.5l_a}{\alpha_M \cdot d}$
α_M	=	1.0 (single curvature) or 2.0 (double curvature) as determined by the designer
l_a	=	effective exposed length considered for bolt bending resistance
	=	$e_1 + a_3$ (see illustrations in Figures 8 and 9)
e_1	=	distance between the concrete surface and the bottom of the leveling nut as pictured in Figures 8 and 9
a_3	=	$0.5d$ where clamping at the concrete surface is not present
	=	0 where clamping at the concrete surface is present

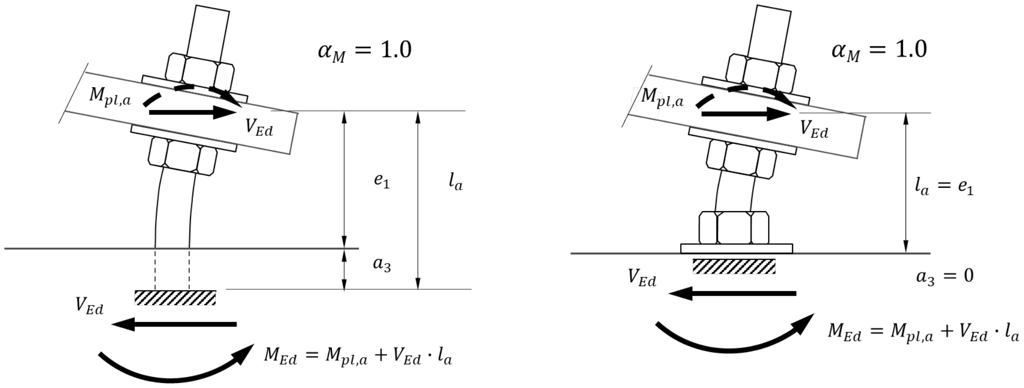

Figure 8. Illustration of dimensions for anchors in single curvature: without clamping nut (left) and with clamping nut (right)

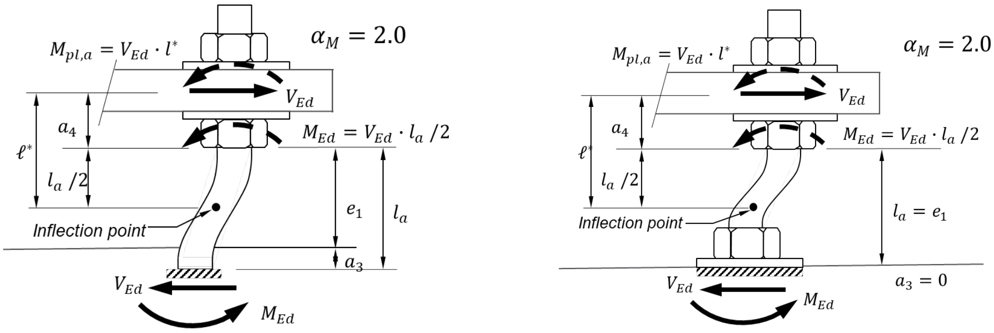

Figure 9. Illustration of dimensions for anchors in double curvature: without clamping nut (left) and with clamping nut (right)

2.3.3 Interaction of steel failure modes

After converting $V_{Rk,s,M}$ and $N_{Rk,s}$ to design values $V_{Rd,s,M}$, and $N_{Rd,s}$ in accordance with EN 1992-4 Table 7.1, the interaction of steel shear and tensile forces is determined in as follows for the Hilti SOFA Method:

$$\left(\frac{N_{Ed}}{N_{Rd,s}}\right)^2 + \frac{V_{Ed}}{V_{Rd,s,M}} \leq 1.0 \qquad (4)$$

The interaction of shear and normal forces in EN 1992-4 design is considered implicitly in EN 1992-4 Eq. (7.37) as given in Eq. (1) of this document.

2.3.4 Concrete failure modes in tension

Tensile concrete failure modes described in EN 1992-4 7.2.1 (cone, pull-out, combined pull-out and concrete, concrete splitting, and concrete blow-out failure) are determined for ungrouted stand-off connections in the same manner as for other connections without modification.

2.3.5 Concrete failure modes in shear

Shear pryout capacity of grouted stand-off connections remains identical to that in EN 1992-4 Section 7.2.2.4.

Shear breakout resistances of ungrouted stand-off connections remain identical to those in EN 1992-4, 7.2.2.5 with the multiplier $\psi_{b,u}$ as given in Eq. (5) on the resistances in EN 1992-4 Eq. (7.40) to account for the bending forces transmitted through the anchor bolt to the concrete.

$$\psi_{b,u} = \frac{1}{1 + \frac{C}{d^{3/4}} \cdot \frac{l_a}{\alpha_M}} \qquad (5)$$

where

C = a constant representing the elastic interaction between the anchor and concrete

$= 0.213$ for ungrouted connections and carries units of $1/mm^{0.25}$

l_a = effective exposed length expressed in mm

α_M = curvature coefficient for the anchor

= 1.0 or 2.0 depending on the assumed curvature (refer to Figures 8 & 9)

2.3.6 Interaction of shear and axial forces for concrete failure

Interaction between tension and shear concrete failure modes per EN 1992-4 Table 7.3 and shall satisfy either Eq. (7.55) or Eq. (7.56). Where supplementary reinforcement is present, EN 1992-4 Section 7.2.3.2 applies.

3. PROFIS ENGINEERING FUNCTIONALITY

Within the Hilti PROFIS Engineering software concrete fixing module, stand-off functionality (2) can be found in the base plate tab (1), as shown in Fig. 10. When stand-off without clamping or clamping can be selected (3), the default restraint level is assumed to be:

- $\alpha_M = 1$ when standoff without clamping is selected
- $\alpha_M = 2$ when standoff with clamping is selected

The user can modify this value (4).

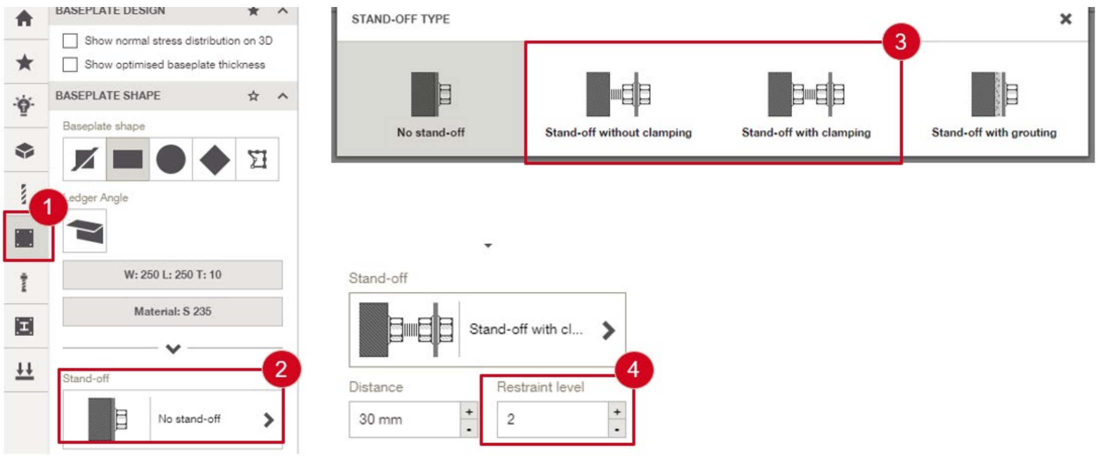

Figure 10. Clamping and restraint options in PROFIS Engineering.

Within the loads tab (1) there are several options for design, and by default the standoff is verified to EN 1992-4 ((2), Fig. 11).

To proceed with the SOFA standoff design method, select this from the standoff section (3).

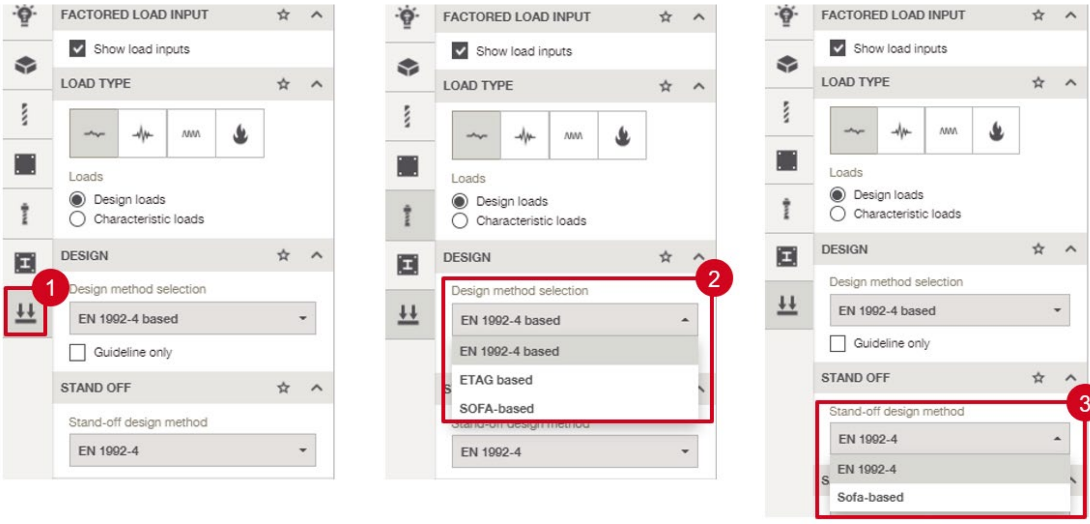

Figure 11. Choice of design method in PROFIS Engineering.

4. DESIGN EXAMPLE

Problem statement:

1. Verify the steel resistance of the anchors in the stand-off base plate connection below for both EN 1992-4 design and for Hilti SOFA Method design, where it has been verified that the conditions for $\alpha_M = 2.0$ from Sec. 2.2.1 have been met.
2. Determine the value of the multiplication factor $\psi_{b,u}$ to be applied to shear concrete edge breakout resistance.

Given:

- Hilti HIT-RE 500 V4 adhesive anchor with Grade 8.8 M24 threaded rod

d	$= 24\ mm$	
A_s	$= 352.7\ mm^2$	
f_{uk}	$= 800\ N/mm^2$	
$\gamma_{Ms,N}$	$= 1.5$	ETA-20/0541 Table C.1
$\gamma_{Ms,V}$	$= 1.25$	ETA-20/0541 Table C.7

- Connection meets the requirements for double curvature (see Sec. 2.2.1)

$\alpha_M \quad = 2.0$

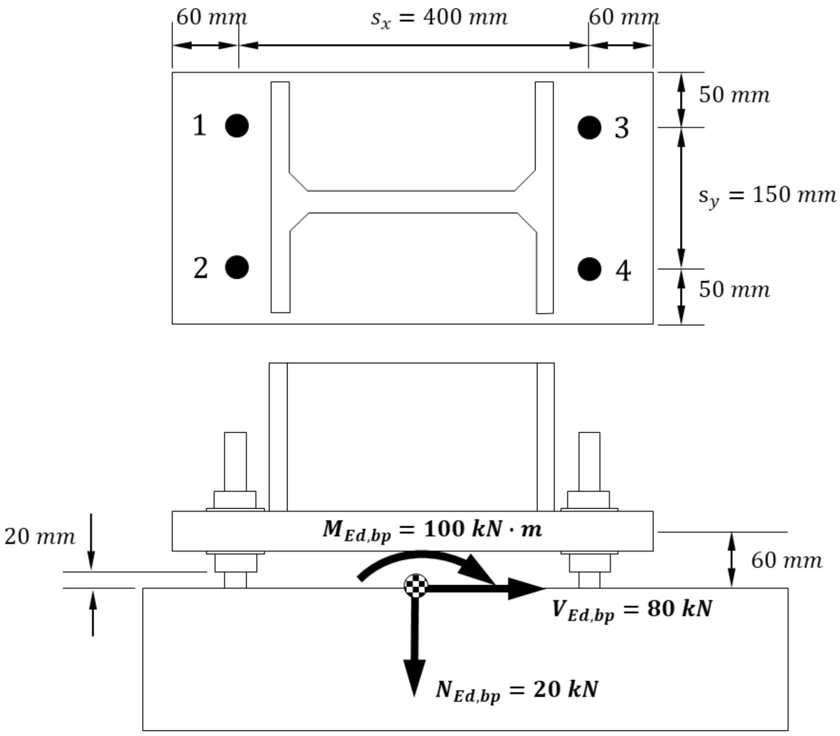

Figure 12. Design example parameters.

Normal forces on anchors:

$$N_{Ed} = -\left(\frac{1}{2}\frac{M_{Ed,bp}}{s_x}\right) + \frac{N_{Ed,bp}}{4} = -\left(\frac{1}{2}\frac{100\ kN\cdot m}{0.4\ m}\right) + \frac{20\ kN}{4} = 120\ kN$$

Anchors 1 and 2 acc. To Fig. 13

$$N_{Ed} = \frac{1}{2}\frac{M_{Ed,bp}}{s_x} + \frac{N_{Ed,bp}}{4} = -\frac{1}{2}\frac{100\ kN\cdot m}{0.4\ m} + \frac{20\ kN}{4} = 130\ kN$$

Anchors 3 and 4 acc. To Fig. 13

Shear forces on anchors:

$$V_{Ed} = \frac{V_{Ed,bp}}{4} = \frac{80\ kN}{4} = 20\ kN$$

All anchors acc. To Fig. 13

Summary of loads by anchor:

Anchor	N_{Ed} (kN)	V_{Ed} (kN)
1	-120	20
2	-120	20
3	130	20
4	130	20

Calculation of basic threaded rod steel resistances:

$N_{Rk,s}$	= characteristic tensile resistance	
	$= A_s \cdot f_{uk}$	ETA-20/0541 Table C1
	$= 352.7\ mm^2 \cdot 800\ N/mm^2$	
	$= 282\ kN$	
$N_{Rd,s}$	$= N_{Rk,s}/\gamma_{Ms,N}$	EN 1992-4 Table 7.1
	$= 282\ kN/1.5$	
	$= 188\ kN$	
k_6	$= 0.5$	EN 1992-4 Sec. 7.2.2.3.1 (1)
$V^0_{Rk,s}$	$= k_6 \cdot A_s \cdot f_{uk}$	EN 1992-4 Eq. (7.34)
	$= 0.5 \cdot 352.7\ mm^2 \cdot 800\ N/mm^2$	
	$= 141\ kN$	
k_7	$= 1.0$	ETA-20/0541 Table C7
$V_{Rk,s}$	$= k_7 \cdot V^0_{Rk,s}$	EN 1992-4 Eq. (7.35)
	$= 1.0 \cdot 141\ kN$	
	$= 141\ kN$	
d_{nt}	= diameter associated with net tensile area	
	$= \sqrt{\frac{4A_s}{\pi}} = \sqrt{\frac{4 \cdot 352.7\ mm^2}{\pi}}$	
	$= 21.2\ mm$	
W_{el}	$= \frac{\pi \cdot d_{nt}^3}{32} = \frac{\pi \cdot 21.2^3}{32}$	
	$= 934.3\ mm^3$	
$M^0_{Rk,s}$	$= 1.2 \cdot W_{el} \cdot f_{uk}$	ETA-20/0541
	$= 1.2 \cdot 934.3\ mm^3 \cdot 800\ N/mm^2$	
	$= 897\ N \cdot m$	

EN 1992-4 Design:

e_1	$= 60\ mm$	EN 1992-4 Fig. 6.6 (a)
a_3	$= \frac{d}{2}$	EN 1992-4 Eq. (6.2)
	$= \frac{24\ mm}{2}$	
	$= 12\ mm$	
l_a	$= e_1 + a_3$	EN 1992-4 Eq. (6.2)
	$= 60\ mm + 12\ mm$	
	$= 72\ mm$	

$M_{Rk,s}$	$= M^0_{Rk,s}\left(1 - \frac{N_{Ed}}{N_{Rd,s}}\right)$	EN 1992-4 Eq. (7.38)
	$= 897\ N \cdot m\left(1 - \frac{\lvert -120\ kN \rvert}{188\ kN}\right)$	Anchors 1 and 2 (see. Fig. 13)
	$= 325\ N \cdot m$	
	$= 897\ N \cdot m\left(1 - \frac{130\ kN}{188\ kN}\right)$	Anchors 3 and 4 (see. Fig. 13)
	$= 277\ N \cdot m$	
$V_{Rk,s,M}$	$= \frac{\alpha_M \cdot M_{Rk,s}}{l_a}$	EN 1992-4 Eq. (7.37)
	$= \frac{2.0 \cdot 325\ N \cdot m}{70\ mm}$	Anchors 1 and 2 (see. Fig. 13)
	$= 9.0\ kN$	
	$= \frac{2.0 \cdot 325\ N \cdot m}{70\ mm}$	Anchors 3 and 4 (see. Fig. 13)
	$= 7.7\ kN$	
$V_{Rd,s,M}$	$= V_{Rk,s,M}/\gamma_{Ms,V}$	EN 1992-4 Table 7.2
	$= 9.0\ kN/1.25$	Anchors 1 and 2 (see. Fig. 13)
	$= 7.2\ kN$	
	$= 7.7\ kN/1.25$	Anchors 3 and 4 (see. Fig. 13)
	$= 6.2\ kN$	
$\beta_{N,V,1/2}$	$= \frac{V_{Ed}}{V_{Rk,s,M}}$	Utilization for EN 1992-4
	$= \frac{20\ kN}{7.2\ kN}$	Anchors 1 and 2 (see. Fig. 13)
	= 277%	Not suitable
$\beta_{N,V,3/4}$	$= \frac{20\ kN}{6.2\ kN}$	Anchors 3 and 4 (see. Fig. 13)
	= 325%	Not suitable

Note that for EN 1992-4 design, $\beta_{N,V}$ is equivalent to β_V, as the interaction of tensile and shear forces is incorporated directly into the calculation of $V_{Rd,s,M}$.

Hilti SOFA Method Design:

e_1	$= 20\ mm$	See Figure 9
a_3	$= \frac{d}{2}$	See Figure 9
	$= \frac{24mm}{2}$	
	$= 12\ mm$	
l_a	$= e_1 + a_3$	See Figure 9
	$= 20\ mm + 12\ mm$	
	$= 32\ mm$	
$\alpha_{s,M}$	$= \frac{1.5 \cdot l_a}{\alpha_M \cdot d}$	See Equation (3)

$= \frac{1.5 \cdot 30\ mm}{2.0 \cdot 24\ mm}$

$= 1.0$

$V_{Rk,s,M}$	$= \left(\sqrt{{\alpha_{s,M}}^2 + 1} - \alpha_{s,M}\right) \cdot V_{Rk,s}$	Equation (3)
	$= \left(\sqrt{1^2 + 1} - 1\right) \cdot 141\ kN$	All anchors (see. Fig. 13)
	$= 58.4\ kN$	
$V_{Rd,s,M}$	$= V_{Rk,s,M}/\gamma_{Ms,V}$	EN 1992-4 Table 7.2
	$= 58.4\ kN/1.25$	All anchors (see. Fig. 13)
	$= 46.8\ kN$	
$\beta_{N,V,1/2}$	$= \left(\frac{N_{Ed}}{N_{Rd,s}}\right)^2 + \frac{V_{Ed}}{V_{Rd,s,M}}$	SOFA interaction utilization; Equation (4)
	$= \left(\frac{\lvert -120\ kN \rvert}{188\ kN}\right)^2 + \frac{20\ kN}{46.8\ kN}$	Anchors 1 and 2 (see. Fig. 13)
	= 84%	Okay
$\beta_{N,V,3/4}$	$= \left(\frac{130\ kN}{188\ kN}\right)^2 + \frac{20\ kN}{46.8\ kN}$	Anchors 3 and 4 (see. Fig. 13)
	= 91%	Okay

Summary of design utilizations for EN 1992-4 and Hilti SOFA

Anchor	N_{Ed} (kN)	V_{Ed} (kN)	β_{EN}	β_{SOFA}
1	-120	20	277%	84%
2	-120	20	277%	84%
3	130	20	325%	91%
4	130	20	325%	91%

Summary of tensile design capacities and utilizations for EN 1992-4 and Hilti SOFA

	Both EN and SOFA		
Anchor	N_{Ed} (kN)	N_{Ed} (kN)	β_N (%)
1	-120	188	64%
2	-120	188	64%
3	130	188	69%
4	130	188	69%

Summary of shear design capacities and utilizations for EN 1992-4 and Hilti SOFA

	Both	EN 1992-4		SOFA	
Anchor	V_{Ed} (kN)	$V_{Rd,s,M}$ (kN)	β_V (%)	$V_{Rd,s,M}$ (kN)	β_V (%)
1	20	7.2	277%	46.8	43%
2	20	7.2	277%	46.8	43%
3	20	6.2	325%	46.8	43%
4	20	6.2	325%	46.8	43%

Summary of interaction design utilizations for EN 1992-4 and Hilti SOFA

	EN*	SOFA		
Anchor	$\beta_{N,V}$ (EN)	β_N (%)	β_V (%)	$\beta_{N,V,}$ (SOFA)
1	277%	74%	22%	84%
2	277%	74%	22%	84%
3	325%	n/a	22%	91%
4	325%	n/a	22%	91%

*The combined tensile and shear utilization for EN 1992-4 is incorporated directly into the equation for shear resistance in EN 1992-4 Eq. (7.37). Therefore, the shear utilizations and the combined utilizations are identical.

Calculation of multiplier $\psi_{b,u}$

The multiplier $\psi_{b,g}$ is automatically applied to the shear breakout failure mode in the Hilti SOFA Method design.

$$\psi_{b,u} = \frac{1}{1+\frac{c}{d^{3/4}}\cdot\frac{l_a}{\alpha_M}} \qquad \text{Equation (5)}$$

$$= \frac{1}{1+\frac{0.213/mm^{-1/4}}{(24\,mm)^{3/4}}\frac{32\,mm}{2}}$$

$$= 0.76$$

The shear breakout resistance will be multiplied by 0.76 in SOFA design.

5. REFERENCES

[1] EN 1992-4 (2018). *Eurocode 2 – Design of concrete structures - Part 4: Design of fastenings for use in concrete*.

[2] fib (2011). *Bulletin 58 – Design of anchorages in concrete: Guide to good practice*.

[3] McBride, K. (2014). *Steel strength of anchor bolts in stand-off base plate connections*. Ph.D. Dissertation, University of Florida, Gainesville, FL, USA.

[4] Scheer, J., Peil, U., and Nölle, P. (1987). *Anchors under planned bending (in German)*. Institut für Stahlbau, Braunschweig, DE.

APPENDIX A: DERIVATION OF REDUCTION FACTOR OF CONCRETE SHEAR FAILURE DUE TO ANCHOR BENDING

Anchor bending moments due to stand-off exacerbate bearing stresses between the anchor and the concrete, which effectively acts as a load amplification factor for concrete edge breakout. In Figure 6, the bending demand due to a shear force is schematically shown on a displaced stand-off anchor. For clarity, additional forces and the deformed shape of the anchor within the concrete are not shown.

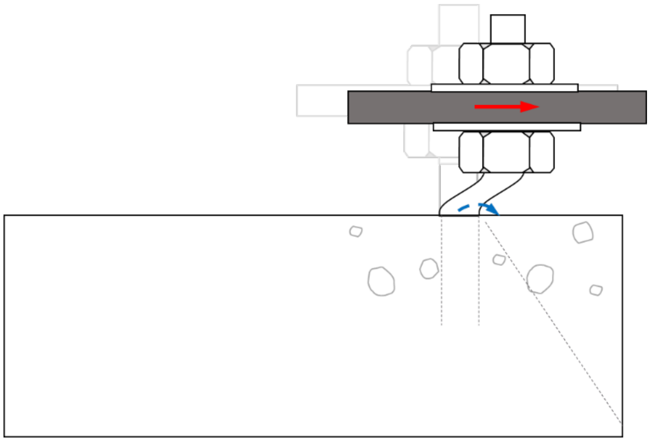

Figure 6. Forces acting on exposed portion of anchor (double curvature shown)

Bearing stresses within concrete due to shear loading with and without a surface moment have been studied extensively in the field of concrete dowel research. In seminal work using the assumption of a semi-infinite beam on an elastic foundation given by Timoshenko and Lessells (1925)[1], Friberg (1940)[2] characterized the deflection of a cylindrical dowel in concrete subjected to shear and bending moment along the length of the dowel.

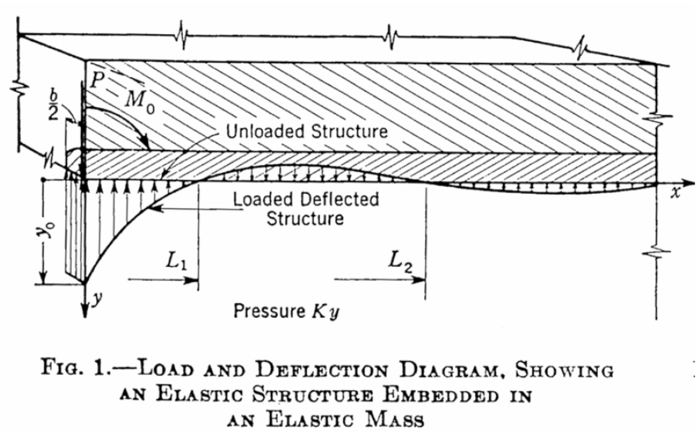

Figure 7. Bearing forces due to shear and moment From Friberg (1940)

The maximum deflection, y_o, is experienced at the surface of the concrete ($x = 0$) and is given as:

[1] Timoshenko, S. and Lessells, J. M. (1925). Applied Elasticity. Westinghouse Technical Night School Press.
[2] Friberg, B. F. (1940). *Design of dowels in transverse joints of concrete pavements.* Transactions of the American Society of Civil Engineers 105(1): 1076-1095.

$$y_o = \frac{P + \beta M_o}{2\beta^3 E_s I} \tag{A1}$$

where

P = Applied shear load

M_o = Bending moment of the dowel at the surface of the concrete

E_s = Dowel modulus of elasticity

I = Dowel moment of inertia

The relative stiffness between the dowel and the concrete mass, β, is defined as follows:

$$\beta = \sqrt[4]{\frac{Kb}{4E_s I}} \tag{A2}$$

where

K = Modulus of dowel support

b = Diameter of dowel

This model may be directly applied to an anchor in a stand-off base plate connection, where M_o is the moment due to shear on the anchor acting over an anchor's effective exposed length, ℓ_{eff}. For the case of zero moment due to stand-off, Eq. (A1) simplifies to

$$y_{o,M_o=0} = \frac{P}{2\beta^3 E_s I} \tag{A3}$$

The bearing pressure on the concrete with and without an initial moment is the deflection from (A1) and (A3) multiplied by K, resulting in (A4) and (A5), respectively:

$$p_o = K y_o = \frac{K(P + \beta M_o)}{2\beta^3 E_s I} \tag{A4}$$

$$p_{o,M_o=0} = K y_{o,M_o=0} = \frac{KP}{2\beta^3 E_s I} \tag{A5}$$

Define $\alpha_{bearing}$ as (A4) divided by (A5), which can be interpreted as the effect of bending moment on the surface pressure:

$$\alpha_{bearing} = \frac{p_o}{p_{o,M_o=0}} = \frac{K(P + \beta M_o)}{2\beta^3 E_s I} \Big/ \frac{KP}{2\beta^3 E_s I} = \frac{K(P + \beta M_o)}{2\beta^3 E_s I} \frac{2\beta^3 E_s I}{KP} \tag{A6}$$

which immediately simplifies to:

$$\alpha_{bearing} = \frac{P + \beta M_o}{P} = 1 + \frac{\beta M_o}{P}$$

Expanding β with (A2), substituting P with V for clarity, and expanding M_o as $V\ell_{eff}/2$ yields:

$$\alpha_{bearing} = 1 + \sqrt[4]{\frac{Kb}{4E_s I}} \frac{V\ell_{eff}}{2V}$$

$$\alpha_{bearing} = 1 + \sqrt[4]{\frac{Kb}{4E_s I}} \frac{\ell_{eff}}{2}$$

Further substituting b with d for clarity and I with $\frac{\pi d^4}{64}$ yields:

$$\alpha_{bearing} = 1 + \sqrt[4]{\frac{64Kd}{4E_s \pi d^4}} \frac{\ell_{eff}}{2}$$

$$\alpha_{bearing} = 1 - \sqrt[4]{\frac{16K}{E_s \pi d^3}} \frac{\ell_{eff}}{2} = 1 + \sqrt[4]{\frac{16K}{E_s \pi}} \sqrt[4]{\frac{1}{d^3}} \frac{\ell_{eff}}{2}$$

Define new constant C as:

$$C = \sqrt[4]{\frac{16K}{E_s \pi}}$$

To reach the final form of $\alpha_{bearing}$:

$$\alpha_{bearing} = 1 + \frac{C}{d^{3/4}} \frac{\ell_{eff}}{2}$$

Define $\psi_{b,u}$ as the inverse of the bearing amplification factor $\alpha_{bearing}$; $\psi_{b,u}$ can be applied to concrete edge breakout resistance in shear as a reduction factor due to bending.

For bolts in double curvature, the expression is as follows:

$$\psi_{b,u} = \frac{1}{\alpha_{bearing}} = \frac{1}{1 + \frac{C}{d^{3/4}} \frac{\ell_{eff}}{2}} \quad \text{(A7a)}$$

For bolts in single curvature, the expression is as follows:

$$\psi_{b,u} = \frac{1}{1 + \frac{C\ell_{eff}}{d^{3/4}}} \quad \text{(A7b)}$$

K can be taken as $3 \, x \, 10^5$ pci ($81.4 \, N/mm^3$) per Yoder and Witczak (1991)[3] and $E_s = 29000 \, ksi$ ($200{,}000 \, N/mm^2$), resulting in $C = 0.48/in^4$ ($0.213/mm^4$). Incorporating the term α_m accounts for the boundary conditions of the anchor to reach the final form:

$$\psi_{b,u,fractional} = \frac{1}{1 + \frac{0.48\ell_{eff}}{\alpha_m d^{3/4}}} \quad \text{(A7c)}$$

$$\psi_{b,u,metric} = \frac{1}{1 + \frac{0.213\ell_{eff}}{\alpha_m d^{3/4}}} \quad \text{(A7d)}$$

Albertson (1992)[4] demonstrated that for practical purposes, dowels of finite length are explained well by the Friberg (1940) model so long as βL is greater than approximately two. For values of βL, less than two, the bearing stresses from the semi-infinite length assumption become conservative. For the assumptions stated herein, the minimum embedment depth of $4d$ in ACI 318 Chapter 17 anchoring provisions always produces βL greater than two.

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[3] Yoder, E. J., Witczak, M. W. Principles of Pavement Design, Second ed. John Wiley and Sons, Inc., 1991.

[4] Albertson, M. D. (1992). “Fibercomposite and steel pavement dowels.” Master’s thesis, Iowa State University, Ames, IA.

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